Minime, maxime. Inegalități $x \in \mathbb{R}$, $y \in \mathbb{R}$

1) Determinați minimul expresiilor

$E = 8 + x^2$

$(\forall) x \in \mathbb{R} \Rightarrow x^2 \geq 0 \mid +8 \Rightarrow 8 + x^2 \geq 8 + 0 \Rightarrow \underline{8 + x^2 \geq 8} \Rightarrow$

val. minimă este 8 pt. $x = 0$

$F = -3 + y^2$

$(\forall) y \in \mathbb{R} \Rightarrow y^2 \geq 0 \mid +(-3) \Rightarrow (-3) + y^2 \geq 0 + (-3) \Rightarrow$

$\underline{-3 + y^2 \geq -3} \Rightarrow$ val. minimă -3 pt. $y = 0$

$G(x) = x^2 + 22x + 129 = x^2 + 2 \cdot 11 \cdot x + 11^2 + 8 = (x+11)^2 + 8$

$(\forall) x \in \mathbb{R} \Rightarrow (x+11)^2 \geq 0 \mid +8 \Rightarrow (x+11)^2 + 8 \geq 0 + 8 \Rightarrow$

$\underline{(x+11)^2 + 8} \geq 8$, $(\forall) x \in \mathbb{R} \Rightarrow$

$\Rightarrow$ val. minimă $= 8$, pt. $x + 11 = 0 \Rightarrow x = -11$

$H(x) = x^2 - 6x + 80 = x^2 - 2 \cdot 3 \cdot x + 3^2 + 71 = (x-3)^2 + 71$

$(\forall) x \in \mathbb{R} \Rightarrow (x-3)^2 \geq 0 \mid +71 \Rightarrow (x-3)^2 + 71 \geq 71 \Rightarrow$ val. min. $= 71$

pt. $x - 3 = 0 \Rightarrow x = 3$

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② Determinați valoarea maximă a expresiilor:

$E(x) = 21 - x^2$

$(\forall) x \in \mathbb{R} \Rightarrow x^2 \geq 0 \mid \cdot(-1) \Rightarrow -x^2 \leq 0 \mid +21 \Rightarrow 21 - x^2 \leq 0 + 21 \Rightarrow$

$21 - x^2 \leq 21 \Rightarrow$ val. maximă este 21, pt. $x = 0$

$F(x) = 100 - 4x^2 - 4x - 1 = 100 - 1 \cdot \left[(2x)^2 + 2 \cdot 2x + 1^2\right]$

$F(x) = 100 - (2x+1)^2$, $(\forall) x \in \mathbb{R} \Rightarrow (2x+1)^2 \geq 0 \mid \cdot(-1) \Rightarrow -(2x+1)^2 \leq 0 \mid +100$

$100-(2x+1)^2 \leq 100 \Rightarrow$ val. max. 100 pt. $2x+1=0 \Rightarrow x=-\frac{1}{2}$

$G(x)=231-x^2+20x$? $-100=231$? $=231+100$

$G(x)=331-x^2+2\cdot x\cdot 10-100=331-(x^2-2\cdot 10x+10^2)=$

$=331-(x-10)^2$

$(\forall)x\in\mathbb{R} \Rightarrow (x-10)^2\geq 0 \mid\cdot(-1) \Rightarrow -(x-10)^2\leq 0 \mid +331 \Rightarrow 331-(x-10)^2\leq 331$

$\Rightarrow$ val maximă $=331$ pt. $x-10=0 \Rightarrow x=10$

$H(x)=36-4x^2+20x$

$H(x)=36-4x^2+20x=61-(2x)^2+2\cdot 2x\cdot 5-5^2=$

$=61-[(2x)^2-2\cdot 2x\cdot 5+5^2]=61-(2x-5)^2$

$(\forall)x\in\mathbb{R} \Rightarrow (2x-5)^2\geq 0 \mid\cdot(-1) \Rightarrow -(2x-5)^2\leq 0 \mid +61 \Rightarrow$

$61-(2x-5)^2\leq 61 \Rightarrow$ val. max. 61, pt. $2x-5=0 \Rightarrow x=\frac{5}{2}$

Det. val. minimă a expresiilor:

$E(x,y)=x^2+14x+y^2+6y+62$

$E(x,y)=x^2+2\cdot x\cdot 7+7^2+y^2+2\cdot y\cdot 3+3^2+4=$

$=(x+7)^2+(y+3)^2+4$

$(\forall)x\in\mathbb{R} \Rightarrow (x+7)^2\geq 0$

$(\forall)y\in\mathbb{R} \Rightarrow (y+3)^2\geq 0$

$(x+7)^2+(y+3)^2\geq 0 \mid +4$

$\Rightarrow (x+7)^2+(y+3)^2\geq 0 \mid +4$

$(x+7)^2+(y+3)^2+4\geq 4$

$\Rightarrow$ val. minimă $=4$ pt. $x+7=0 \Rightarrow x=-7$ și $y+3=0 \Rightarrow y=-3$

$G(x,y)=25x^2-10x+36y^2+12y+100$

$G(x,y)=(5x-1)^2+(6y+1)^2+98$

$(5x-1)^2\geq 0, (\forall)x\in\mathbb{R}$

$(6y+1)^2\geq 0, (\forall)y\in\mathbb{R}$

$\Rightarrow (5x-1)^2+(6y+1)^2\geq 0 \mid +98 \Rightarrow (5x+1)^2+(6y+1)^2+98\geq 98$

$\Rightarrow$ val. min. 98 pt. $x=\frac{1}{5}$, $y=-\frac{1}{6}$

b) $x \in \mathbb{R}$, $y \in \mathbb{R}$

$x^2 + 4y^2 - 4 \cdot (x + 3y) + 13 = 0$

m_g a lui x si y

$x^2 + 4y^2 - 4(x + 3y) + 13 = 0 \Leftrightarrow$

$x^2 + 4y^2 - 4x - 12y + 13 = 0 \Leftrightarrow$

$x^2 - 2 \cdot 2x + 2^2 + (2y)^2 - 2 \cdot 2 \cdot y \cdot 3 + 3^2 = 0 \Leftrightarrow$

$(x-2)^2 + (2y-3)^2 = 0$

$(\forall) x \in \mathbb{R} \Rightarrow (x-2)^2 \geq 0$

$(\forall) y \in \mathbb{R} \Rightarrow (2y-3)^2 \geq 0$

$\Rightarrow x - 2 = 0$ si $2y - 3 = 0 \Rightarrow$

$x = 2$ $\quad y = \frac{3}{2}$

$m_g = \sqrt{x \cdot y} = \sqrt{2 \cdot \frac{3}{2}} = \sqrt{3}$

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