Inecuații

Rezolvați în $\mathbb{R}$:

① $x\cdot\left(-\frac{1}{1\cdot2}-\frac{1}{2\cdot3}-\frac{1}{3\cdot4}-\dots-\frac{1}{2019\cdot2020}\right)\le-\frac{2019}{2020}$ $\Leftrightarrow$

$x\cdot(-1)\cdot\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{2019\cdot2020}\right)\le-\frac{2019}{2020}\ \Big|\cdot(-1)$

$\Leftrightarrow x\cdot\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{2019\cdot2020}\right)\ge\frac{2019}{2020}$ (*)

$n\in\mathbb{N}^*\Rightarrow$

$\frac{1}{n}-\frac{1}{n+1}=\frac{n+1}{n(n+1)}-\frac{n}{n(n+1)}=\frac{\cancel{n}+1-\cancel{n}}{n(n+1)}=\frac{1}{n(n+1)}$

$\Rightarrow\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}\Rightarrow\frac{1}{1\cdot2}=\frac{1}{1}-\frac{1}{2};\ \frac{1}{2\cdot3}=\frac{1}{2}-\frac{1}{3}$

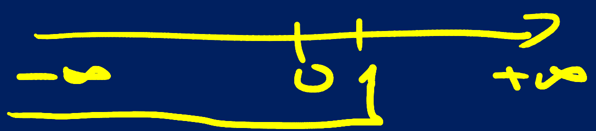
(*) $\Leftrightarrow$

$x\cdot\left(\frac{1}{1}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}+\dots+\cancel{\frac{1}{2019}}-\frac{1}{2020}\right)\ge\frac{2019}{2020}$ $\Leftrightarrow$

$x\cdot\left(\frac{1}{1}-\frac{1}{2020}\right)\ge\frac{2019}{2020}\Leftrightarrow x\cdot\left(\frac{2020}{2020}-\frac{1}{2020}\right)\ge\frac{2019}{2020}\Leftrightarrow$

$x\cdot\frac{2019}{2020}\le\frac{2019}{2020}\Leftrightarrow x\le\frac{2019}{2020}:\frac{2019}{2020}\Leftrightarrow x\le1$

$\rightarrow S=(-\infty,1]$



(2) $\frac{1}{3}\cdot(x+1)-\frac{1}{2}\cdot(x-4)>x \Leftrightarrow$

$^{2/}\frac{x+1}{3}-{}^{3/}\frac{x-4}{2}>{}^{6/}\frac{x}{1} \Leftrightarrow \frac{2x+2}{6}-\frac{3x-12}{6}>\frac{6x}{6} \Leftrightarrow$

$\frac{2x+2-3x+12}{6}>\frac{6x}{6} \Leftrightarrow -x+14>6x \Leftrightarrow$

$-x-6x>-14 \Leftrightarrow -7x>-14 \ \Big|\cdot(-1) \Leftrightarrow$

$7x<14 \Leftrightarrow x<\frac{14}{7} \Leftrightarrow x<2 \Rightarrow S=(-\infty,2)$

(3)

$\frac{x+1}{2}+\frac{2x-5}{-3}\geq 2{,}5 \Leftrightarrow {}^{3/}\frac{x+1}{2}-{}^{2/}\frac{2x-5}{3}\geq{}^{3/}\frac{5}{2} \Leftrightarrow$

$\frac{3x+3}{6}-\frac{4x-10}{6}\geq\frac{15}{6} \Leftrightarrow \frac{3x+3-4x+10}{6}\geq\frac{15}{6} \Leftrightarrow$

$-x+13\geq 15 \Leftrightarrow -x\geq 15-13 \Leftrightarrow -x\geq 2 \ \Big|\cdot(-1)$

$x\leq -2 \Rightarrow S=(-\infty,-2]$

$\sqrt{18}=\sqrt{9\cdot 2}=\sqrt{9}\cdot\sqrt{2}=3\sqrt{2}$

$-1-3=-4$

(4)

$2\cdot(1+x\sqrt{2})+\sqrt{18}\leq 3x+8-\sqrt{2} \Leftrightarrow$

$2+2\sqrt{2}\cdot x+3\sqrt{2}\leq 3x+8-\sqrt{2} \Leftrightarrow$

$2\sqrt{2}x-3x\leq 8-\sqrt{2}-2-3\sqrt{2} \Leftrightarrow$

$x\cdot(2\sqrt{2}-3)\leq 6-4\sqrt{2} \Leftrightarrow$

$x\cdot(2\sqrt{2}-3)\leq \underbrace{2\cdot 3-2}\cdot 2\sqrt{2} \Leftrightarrow$

$$x\cdot(2\sqrt{2}-3)\le -2\cdot(2\sqrt{2}-3) \Leftrightarrow$$

$$x \le \frac{-2\cancel{(2\sqrt{2}-3)}}{\cancel{(2\sqrt{2}-3)}\cdot 1} \Leftrightarrow x \le -2 \Rightarrow \quad ???$$

$S=(-\infty,-2]$, deoarece

ASA NU!

nu am studiat semnul nr. $2\sqrt{2}-3$? 0

Studiem semnul nr. $2\sqrt{2}-3$.

$$\left.\begin{array}{l} 2\sqrt{2}=\sqrt{4}\cdot\sqrt{2}=\sqrt{8} \\ 3=\sqrt{9} \end{array}\right\} \begin{array}{l} 8<9 \Rightarrow \sqrt{8}<\sqrt{9} \Rightarrow 2\sqrt{2}<3 \Rightarrow \\ 2\sqrt{2}-3<0 \end{array}$$

$$x\cdot(2\sqrt{2}-3)\le -2(2\sqrt{2}-3) \;\Big|\cdot \frac{1}{2\sqrt{2}-3} \Leftrightarrow$$

$$x\cdot\cancel{(2\sqrt{2}-3)}^{1}\cdot\frac{1}{\cancel{(2\sqrt{2}-3)}_{1}\cdot 1} \ge -2\cdot\cancel{(2\sqrt{2}-3)}^{1}\cdot\frac{1}{\cancel{(2\sqrt{2}-3)}_{1}\cdot 1}$$

$$x \ge -2 \Rightarrow S=[-2;+\infty)$$

OBSERVAŢII:

$$\underbrace{\frac{3}{4}=\frac{1+\cancel{2}^{1}}{\cancel{2}_{1}\cdot 2}}_{\text{BUN}} \ne \underbrace{\frac{2}{2}=1}_{\text{BUN}}$$

$x \in \mathbb{R} \setminus \{-1\}$, $\dfrac{x+1}{3x+3} = \dfrac{x+1}{3(x+1)} = \dfrac{1 \cdot \cancel{(x+1)}^{1}}{3 \cdot \cancel{(x+1)}_{1}} = \dfrac{1}{3}$

$\dfrac{\cancel{2}^{1} \cdot 3}{\cancel{2}_{1}} = \dfrac{3}{1}$ $\dfrac{\cancel{3}^{1} \cdot \cancel{8}^{1}}{\cancel{3}_{1} \cdot \cancel{16}_{2}} = \dfrac{1}{2}$

$(-10) : 2 = 10 : (-2) = -(10 : 2)$

$\dfrac{-10}{2} = \dfrac{10}{-2} = -\dfrac{10}{2}$

$\dfrac{2x-3}{-3} = -\dfrac{2x-3}{3} = \dfrac{-(2x-3)}{3} = \dfrac{-2x+3}{3} = \dfrac{3-2x}{3}$

<u>OBS</u>: când înmulțim o inecuație cu un nr. nenul, studiem semnul acestuia.

$x \cdot 2\sqrt{3} + 5 \leq x \cdot 3\sqrt{2} \Leftrightarrow$

$x \cdot 2\sqrt{3} - x \cdot 3\sqrt{2} \leq -5 \Leftrightarrow$

$x \cdot (2\sqrt{3} - 3\sqrt{2}) \leq -5$

Studiem semnul nr. $2\sqrt{3} - 3\sqrt{2}$:

$$\left.\begin{array}{l} 2\sqrt{3} = \sqrt{4} \cdot \sqrt{3} = \sqrt{4 \cdot 3} = \sqrt{12} \\ 3\sqrt{2} = \sqrt{9} \cdot \sqrt{2} = \sqrt{9 \cdot 2} = \sqrt{18} \end{array}\right\} \begin{array}{l} 12 < 18 \Rightarrow \sqrt{12} < \sqrt{18} \Rightarrow \\ 2\sqrt{3} < 3\sqrt{2} \Rightarrow \\ 2\sqrt{3} - 3\sqrt{2} < 0 \end{array}$$

$$x\cdot(2\sqrt{3}-3\sqrt{2})\le -5 \;\Big|\cdot\frac{1}{2\sqrt{3}-3\sqrt{2}}$$

$$x\cdot\cancelto{1}{(2\sqrt{3}-3\sqrt{2})}\cdot\frac{1}{1\cdot\cancelto{1}{(2\sqrt{3}-3\sqrt{2})}}\ge -5\cdot\frac{1}{2\sqrt{3}-3\sqrt{2}}$$

$$x\ge\frac{-5}{2\sqrt{3}-3\sqrt{2}}\iff x\ge-\frac{5}{2\sqrt{3}-3\sqrt{2}}\iff$$

$$x\ge\frac{5}{-(2\sqrt{3}-3\sqrt{2})}\iff x\ge\frac{{}^{3\sqrt{2}+2\sqrt{3})}5}{3\sqrt{2}-2\sqrt{3}}\iff$$

$$(a+b)\cdot(a-b)=a^2-b^2$$

$$x\ge\frac{5\cdot(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})\cdot(3\sqrt{2}+2\sqrt{3})}$$

$$x\ge\frac{5\cdot(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}\iff x\ge\frac{5\cdot(3\sqrt{2}+2\sqrt{3})}{18-12}$$

$$x\ge\frac{15\sqrt{2}+10\sqrt{3}}{6}\Rightarrow S=\left[\frac{15\sqrt{2}+10\sqrt{3}}{6},+\infty\right)$$

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