Expresii: aduceți la forma cea mai simplă:

$$E(x)=\frac{x}{x^2-25}-\left[\frac{3}{x-5}-\left(\frac{x+1}{x-5}-\frac{3}{x+5}+\frac{x^2}{25-x^2}\right)\right]$$

R.c. $x^2-25\neq 0 \Leftrightarrow (x-5)(x+5)\neq 0 \Leftrightarrow x-5\neq 0$ și $x+5\neq 0$; $x\neq 5$, $x\neq -5$ $\Rightarrow$

$x\in\mathbb{R}\setminus\{5;-5\}$

Știm că $-8:(+2)=-(8:2)=8:(-2)=-4 \Rightarrow$

$$\frac{-8}{2}=-\frac{8}{2}=\frac{8}{-2}=-4$$

$$\frac{x^2}{25-x^2}=\frac{x^2}{-1\cdot(-25+x^2)}=\frac{x^2}{-(x^2-25)}=-\frac{x^2}{(x-5)(x+5)}$$

$$E(x)=\frac{x}{(x-5)(x+5)}-\left[{}^{x+5)}\frac{3}{x-5}-\left({}^{x+5)}\frac{x+1}{x-5}-{}^{x-5)}\frac{3}{x+5}-\frac{x^2}{(x-5)(x+5)}\right)\right]$$

$$E(x)=\frac{x}{(x-5)(x+5)}-\left[\frac{3x+15}{(x-5)(x+5)}-\frac{x^2+x+5x+5}{(x-5)(x+5)}+\frac{3x-15}{(x-5)(x+5)}+\frac{x^2}{(x-5)(x+5)}\right]$$

$$E(x)=\frac{x}{(x-5)(x+5)}-\frac{3x+15}{(x-5)(x+5)}+\frac{x^2+6x+5}{(x-5)(x+5)}-\frac{3x-15}{(x-5)(x+5)}-\frac{x^2}{(x-5)(x+5)}$$

$$E(x)=\frac{x-\cancel{3x}-\cancel{15}+\cancel{x^2}+\cancel{6x}+5-\cancel{3x}+\cancel{15}-\cancel{x^2}}{(x-5)(x+5)}$$

$$E(x)=\frac{\cancel{x+5}^{\,1}}{(x-5)\cancel{(x+5)}}, \Rightarrow E(x)=\frac{1}{x-5},\ x\in\mathbb{R}\setminus\{5;-5\}$$

$$F(x)=\left[\left(\frac{2x}{3x+1}\right)^7\right]^3 : \left[\frac{4x^2}{(3x+1)^2}\right]^{10}\cdot\frac{3x+1}{2x}$$

P.c. $3x+1\neq 0 \Leftrightarrow 3x\neq -1 \Leftrightarrow x\neq -\frac{1}{3}$; $x^2\neq 0 \Rightarrow x\neq 0$ $\Rightarrow x\in\mathbb{R}^*\setminus\left\{-\frac{1}{3}\right\}$

$$F(x)=\left(\frac{2x}{3x+1}\right)^{21} : \left[\left(\frac{2x}{3x+1}\right)^2\right]^{10}\cdot\frac{3x+1}{2x}=\left(\frac{2x}{3x+1}\right)^{21} : \left(\frac{2x}{3x+1}\right)^{20}\cdot\frac{3x+1}{2x}$$

$$F(x)=\frac{\cancel{2x}}{\cancel{3x+1}}\cdot\frac{\cancel{3x+1}}{\cancel{2x}} \Rightarrow F(x)=1,\ x\in\mathbb{R}^*\setminus\left\{-\frac{1}{3}\right\}$$

$$\frac{\frac{1}{2}}{\frac{1}{3}}=\frac{1}{2}:\frac{1}{3}=\frac{1}{2}\cdot\frac{3}{1}=\frac{3}{2}$$

$$\frac{\frac{\frac{2}{3}}{4}}{\frac{5}{6}}=\frac{\frac{\frac{2}{3}}{\frac{4}{1}}}{\frac{5}{6}}=\frac{\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{4}}}{\frac{5}{6}}=\frac{\frac{1}{6}}{\frac{5}{6}}=\frac{1}{\cancel{6}}\cdot\frac{\cancel{6}}{5}=\frac{1}{5}$$

$$E(x)=\frac{1}{1-\frac{1}{1-\frac{1}{1-\frac{1}{x+1}}}}$$

P.c. $x+1\neq 0 \Leftrightarrow x\neq -1$

$$\frac{1}{1}-\frac{1}{x+1}\neq 0 \Leftrightarrow \frac{x+1}{x+1}-\frac{1}{x+1}\neq 0 \Leftrightarrow$$

$$\frac{x+1-1}{x+1}\neq 0 \Leftrightarrow \frac{x}{x+1}\neq 0 \Leftrightarrow x\neq 0 \text{ (dar } x\neq -1)$$

$$1-\frac{1}{1-\frac{1}{x+1}}\neq 0 \Leftrightarrow 1-\frac{1}{\frac{x}{x+1}}\neq 0 \Leftrightarrow$$

$$\frac{1}{1}-\frac{x+1}{x}=\frac{x-x-1}{x}=-\frac{1}{x}\neq 0 \text{ deoarece } x\neq 0$$

$$1-\frac{1}{1-\frac{1}{1-\frac{1}{1-\frac{1}{x+1}}}}\neq 0 \Leftrightarrow$$

$$1-\frac{1}{-\frac{1}{x}}\neq 0 \Leftrightarrow 1+x\neq 0 \Leftrightarrow x\neq -1 \Rightarrow x\in\mathbb{R}^*\setminus\{-1\}$$

$$E(x) = \frac{1}{x+1},\ x \in \mathbb{R}^* \setminus \{-1\}$$

$$F(x) = \left(\frac{1}{x+3}^{(x-3} + \frac{x}{x-3}^{(x+3} - \frac{1}{1}^{(x-3)(x+3)} \right) \cdot \frac{x+3}{12x+18}$$

P.C. $x+3 \neq 0 \Rightarrow x \neq -3$ $\quad x-3 \neq 0 \Rightarrow x \neq 3$

$12x+18 \neq 0 \Leftrightarrow 6\cdot(2x+3) \neq 0 \Leftrightarrow 2x+3 \neq 0 \Leftrightarrow x \neq -\frac{3}{2}$

$\Rightarrow x \in \mathbb{R} \setminus \{3; -\frac{3}{2}\}$

$$F(x) = \left[\frac{x-3}{(x+3)(x-3)} + \frac{x^2+3x}{(x-3)(x+3)} - \frac{x^2-9}{(x-3)(x+3)} \right] \cdot \frac{x+3}{6(2x+3)}$$

$$F(x) = \frac{x-3+\cancel{x^2}+3x-\cancel{x^2}+9}{(x-3)\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{6(2x+3)} = \frac{4x+6}{x-3} \cdot \frac{1}{6(2x+3)}$$

$$F(x) = \frac{\cancel{2}\cancel{(2x+3)}}{x-3} \cdot \frac{1}{\cancel{6}\cancel{(2x+3)}} = \frac{1}{3(x-3)}$$

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