Expresii

Simplificați expresiile:

$$E(x)=\frac{x^2+x-6}{x^2+2x-8}$$

Punem cond. (p. c.)

$x^2+2x-8\neq 0 \Leftrightarrow x^2+4x-2x-8\neq 0 \Leftrightarrow$

$\Leftrightarrow x(x+4)-2\cdot(x+4)\neq 0 \Leftrightarrow (x+4)(x-2)\neq 0 \Leftrightarrow$

$x+4\neq 0$ și $x-2\neq 0 \Leftrightarrow x\in\mathbb{R}\setminus\{-4;2\}$

$x\neq -4 \quad x\neq 2$

$$E(x)=\frac{(x+3)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$$

$x^2+x-6=x^2+3x-2x-6=x(x+3)-2\cdot(x+3)=$
$=(x+3)(x-2)$

$$\Rightarrow E(x)=\frac{x+3}{x+4}$$

$$F(x)=\frac{x^2-8x+15}{x^2-2x-15}$$

$x^2-2x-15\neq 0 \Leftrightarrow x^2+3x-5x-15\neq 0 \Leftrightarrow$

$\Leftrightarrow x(x+3)-5(x+3)\neq 0 \Leftrightarrow (x+3)(x-5)\neq 0$

$\Leftrightarrow x+3\neq 0$ și $x-5\neq 0 \Leftrightarrow x\neq -3$ și $x\neq 5$

$x\in\mathbb{R}\setminus\{-3;5\}$

$x^2-8x+15=x^2-5x-3x+15=x(x-5)-3(x-5)=(x-5)(x-3)$

$$F(x)=\frac{\cancel{(x-5)}(x-3)}{(x+3)\cancel{(x-5)}}=\frac{x-3}{x+3}$$

Rezolvăm ecuațiile

$$H(x)=\frac{x^2+3x-1}{2x^2-2x-1}$$

$2x^2-2x-1=0$
$a=2$
$b=-2$
$c=-1$

$\Delta=b^2-4ac=(-2)^2-4\cdot2\cdot(-1)$

$\Delta=4+8=12$

$\sqrt{\Delta}=\sqrt{12}=2\sqrt{3}$

$1\cdot x^2+3x-1\neq 0$
$a=1$
$b=3$
$c=-1$

$\Delta=b^2-4ac$

$\Delta=3^2-4\cdot1\cdot(-1)$

$\Delta=9+4=13 \quad \sqrt{\Delta}=\sqrt{13}$

$$x_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-(-2)\pm2\sqrt{3}}{2\cdot2}=\frac{2\pm2\sqrt{3}}{4}=\frac{2(1\pm\sqrt{3})}{4}$$

$$x_{1,2}=\begin{cases}\dfrac{1+\sqrt{3}}{2}\\[2mm]\dfrac{1-\sqrt{3}}{2}\end{cases}$$

P.c. $2x^2-2x-1\neq 0 \Leftrightarrow x\in\mathbb{R}\setminus\left\{\frac{1\pm\sqrt{3}}{2}\right\}$

$ax^2+bx+c=a\cdot(x-x_1)(x-x_2),\ a\neq 0$

$$2x^2-2x-1=2\cdot\left(x-\frac{1+\sqrt{3}}{2}\right)\left(x-\frac{1-\sqrt{3}}{2}\right)$$

$$x^2+3x-1 \neq 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{13}}{2a} = \frac{-3 \pm \sqrt{13}}{2} \Rightarrow$$

$$x^2+3x-1 = \left(x - \frac{-3+\sqrt{13}}{2}\right)\left(x - \frac{-3-\sqrt{13}}{2}\right) = \left(x + \frac{3-\sqrt{13}}{2}\right)\left(x + \frac{3+\sqrt{13}}{2}\right)$$

$$H(x) = \frac{\left(x + \frac{3-\sqrt{13}}{2}\right)\left(x + \frac{3+\sqrt{13}}{2}\right)}{2\cdot\left(x - \frac{1-\sqrt{13}}{2}\right)\left(x - \frac{1+\sqrt{13}}{2}\right)} = \frac{\frac{2x+3-\sqrt{13}}{2}\cdot\frac{2x+3+\sqrt{13}}{2}}{\cancel{2}\cdot\frac{2x-1+\sqrt{13}}{\cancel{2}}\cdot\frac{2x-1-\sqrt{13}}{2}}$$

$$H(x) = \frac{(2x+3-\sqrt{13})(2x+3+\sqrt{13})}{2\cdot\cancel{2}}\cdot\frac{\cancel{2}}{(2x-1+\sqrt{13})(2x-1-\sqrt{13})} = \frac{(2x+3-\sqrt{13})(2x+3+\sqrt{13})}{2\cdot(2x-1+\sqrt{13})(2x-1-\sqrt{13})}$$

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