$(x+4)(x-1)-5(x-3)(x+2)=(x+1)^2-(x-3)^2-2 \Leftrightarrow$

$x^2-x+4x-4-5\cdot(x^2+2x-3x-6)=x^2+2x+1-(x^2-6x+9)-2 \Leftrightarrow$

$x^2+3x-4-5\cdot(x^2-x-6)=x^2+2x+1-x^2+6x-9-2 \Leftrightarrow$

$x^2+3x-4-5x^2+5x+30=8x-10 \Leftrightarrow$

$-4x^2+8x+26=8x-10 \Leftrightarrow -4x^2+8x+26-8x+10=0 \Leftrightarrow$

$-4x^2+36=0 \Leftrightarrow -4x^2=-36 \Leftrightarrow x^2=\frac{-36}{-4} \Leftrightarrow x^2=9 \Leftrightarrow$

$\sqrt{x^2}=\sqrt{9} \Leftrightarrow |x|=3 \Rightarrow \begin{cases} x_1=3 \\ \text{sau} \\ x_2=-3 \end{cases} \Rightarrow S=\{-3;3\}$

(VI)

$-4x^2+36=0 \Leftrightarrow -4(x^2-9)=0 \Leftrightarrow -4(x-3)(x+3)=0$

$x-3=0$ sau $x+3=0$

$x=3$ $\quad x=-3$

$(x+5)(x-4)+3(x-3)(x+2)=(x+2)^2+2\cdot(x+9) \Leftrightarrow$

$x^2-4x+5x-20+3(x^2+2x-3x-6)=x^2+4x+4+2x+18 \Leftrightarrow$

$x^2+x-20+3\cdot(x^2-x-6)=x^2+6x+22 \Leftrightarrow$

$x^2+x-20+3x^2-3x-18-x^2-6x-22=0 \Leftrightarrow$

$3x^2-8x-60=0$

$a=3 \quad b=-8 \quad c=-60$

$\Rightarrow \Delta=b^2-4ac=(-8)^2-4\cdot 3\cdot(-60)$

$\Delta=64+720=784$

$\sqrt{\Delta}=\sqrt{784}=28$

$\sqrt{784}$ | 28

4 | $48\cdot 8=384$

384

384

$x_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-(-8)\pm 28}{2\cdot 3}=\frac{8\pm 28}{6}$

$x_1=\frac{8+28}{6}=\frac{36}{6}=6 \qquad x_2=\frac{8-28}{6}=\frac{-20}{6}=-\frac{10}{3}$

$S=\{6;-\frac{10}{3}\}$

$3x^2-8x-60=3\cdot\left(x-\left(-\frac{10}{3}\right)\right)\cdot(x-6)=(3x+10)(x-6)$

$(x-2)^2+8=0$

$mx^2+3x-1=0$

$m=?$ a.î. $S_1=S_2$

$(x-2)^2\geq 0$, $(\forall)x\in\mathbb{R}\Rightarrow$

$(x-2)^2+8>0$, $(\forall)x\in\mathbb{R}\Rightarrow$

$(x-2)^2+8=0$ imposibil, $(\forall)x\in\mathbb{R}$

$\Downarrow$

$S_1=\Phi$

$S_1=S_2$ dacă și numai dacă $(x-2)^2+8=0 \Leftrightarrow mx^2+3x-1=0$

$a=m \quad b=3 \quad c=-1$

$S_2=S_1=\Phi \Rightarrow \Delta<0$

$\Delta=b^2-4ac=9-4m\cdot(-1)=9+4m$ $\Rightarrow 4m+9<0\Leftrightarrow$

$4m<-9 \Leftrightarrow$

$m<-\frac{9}{4}$

R: $S_1=S_2 \Leftrightarrow m\in\left(-\infty,-\frac{9}{4}\right)$

$(x-2)^2+8=3x \quad (1)$

$(m-n)\cdot x^2-(m+n)\cdot x+2m+n+1=0$

$m=?, n=?$ a.î. $(1)\Leftrightarrow(2)$

$(x-2)^2+8=3x \Leftrightarrow x^2-4x+4+8-3x=0\Leftrightarrow x^2-7x+12=0$

$\Delta=b^2-4ac=49-4\cdot 12=49-48=1\Rightarrow\sqrt{\Delta}=1$

$x_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{7\pm 1}{2}=\begin{cases}\frac{8}{2}=4\\ \frac{6}{2}=3\end{cases}\Rightarrow S_1=\{4;3\}$

$(1)\Leftrightarrow(2)$ d.m.d. $S_1=S_2\Rightarrow 4\in S_2$, $3\in S_2$

$(m-n)\cdot 16-(m+n)\cdot 4+2m+n+1=0\Rightarrow$

$(m-n)\cdot 9-(m+n)\cdot 3+2m+n+1=0\Rightarrow$

$16m-16n-4m-4n+2m+n+1=0\Rightarrow$

$9m-9n-3m-3n+2m+n+1=0\Rightarrow$

$14m-19n+1=0 \quad |\cdot 4$

$8m-11\cdot n+1=0 \quad |\cdot(-7)$

$14=7\cdot 2$

$8=2^3$

$[14,8]=7\cdot 2^3=7\cdot 8=56$

$56:14=4$

$56:8=7$

$56m - 76m + 4 = 0$

$-56m + 77m - 7 = 0$ | +

$m - 3 = 0$

$m = 3$

$8m - 11m + 1 = 0 \Rightarrow$

$8m - 11 \cdot 3 + 1 = 0 \Rightarrow$

$8m = 32 \Rightarrow m = \frac{32}{8} \Rightarrow m = 4$

$(m-m)\cdot x^2 - (m+m)\cdot x + 2m + n + 1 = 0 \Rightarrow$

$(4-3)\cdot x^2 - (4+3)\cdot x + 2\cdot 4 + 3 + 1 = 0 \Rightarrow x^2 - 7x + 12 = 0$

$$\frac{3x}{2\cdot(9x+3)} = \frac{x+2}{3x-1} - \frac{8x^2+3}{9x^2-1} \Leftrightarrow$$

$$\frac{3x}{2\cdot 3\cdot(3x+1)} = \frac{x+2}{(3x-1)} - \frac{8x^2+3}{(3x-1)(3x+1)} \Leftrightarrow$$

(3x+1)

$$\frac{x}{2(3x+1)} = \frac{3x^2+6x+x+2-8x^2-3}{(3x-1)(3x+1)} \Leftrightarrow$$

(3x-1)

$$\frac{3x^2-x}{2(3x+1)(3x-1)} = \frac{-5x^2+7x-1}{(3x-1)(3x+1)} \Leftrightarrow \frac{3x^2-x}{2(3x+1)(3x-1)} = \frac{-10x^2+14x-2}{2(3x+1)(3x-1)} \Rightarrow$$

P.C.M. $9x+3 \neq 0 \Leftrightarrow 3(3x+1) \neq 0 \Leftrightarrow 3x+1 \neq 0 \Leftrightarrow x \neq -\frac{1}{3}$

$3x-1 \neq 0 \Rightarrow x \neq \frac{1}{3}$

Deoarece: $3x+1 \neq 0$ și $3x-1 \neq 0 \Rightarrow (3x+1)(3x-1) \neq 0$

$\Downarrow$

$9x^2 - 1 \neq 0$ (A)

$3x^2 - x + 10x^2 - 14x + 2 = 0 \Leftrightarrow 13x^2 - 15x + 2 = 0$

$\Delta = b^2 - 4ac = 225 - 104 = 121 \Rightarrow \sqrt{\Delta} = \sqrt{121} = 11$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-15) \pm 11}{2\cdot 13} = \frac{15 \pm 11}{26} =$ / $\frac{4}{26} = \frac{2}{13}$; \ $\frac{26}{26} = 1$

$S = \left\{\frac{2}{13}; 1\right\}$

$$\frac{(x+3)^2}{5} + 2 - \frac{(3x-1)^2}{5} = 2x(x-1) - x \;\Big|\cdot 5 \quad \Leftrightarrow$$

$$(x+3)^2 + 2\cdot 5 - (3x-1)^2 = 10x(x-1) - 5x \Leftrightarrow$$

$$x^2+6x+9+10-9x^2+6x-1 = 10x^2-10x-5x \Leftrightarrow$$

$$-8x^2+12x+18-10x^2+15x=0 \Leftrightarrow$$

$$-18x^2+27x+18=0 \;\Big|:(-9)$$

$$2x^2-3x-2=0 \qquad \Delta = b^2-4ac = 9+16=25 \Rightarrow \sqrt{\Delta}=5$$

$$x_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2\cdot a} = \frac{3\pm 5}{4} = \begin{cases} \frac{8}{4}=2 \\ -\frac{2}{4}=-\frac{1}{2} \end{cases} \Rightarrow S=\left\{2;-\frac{1}{2}\right\}$$

$$2x^2-3x-2 = 2\cdot\left(x-\left(-\frac{1}{2}\right)\right)\cdot(x-2) = (2x+1)(x-2)$$

$$(a+1)\cdot x^2 + (a^2+2a+2)\cdot x + a + 1 = 0, \quad a\in\mathbb{R}\setminus\{-1\}, \; x\in\mathbb{R}$$

$S\neq\emptyset$, $(\forall)\, a\in\mathbb{R}\setminus\{-1\}$ $\Big\{ \Delta = (a^2+2a+1)^2 - 4\cdot(a+1)\cdot(a+1)$

$S\neq\emptyset \Leftrightarrow \Delta\geq 0 \Leftrightarrow (a^2+2a+2)^2 - 4\cdot(a+1)\cdot(a+1)\geq 0$

$$(a^2+2a+1+1)^2 - 4\cdot(a+1)^2 \geq 0 \Leftrightarrow \left[(a+1)^2+1\right]^2 - 4(a+1)^2 \geq 0$$

Notăm: $a+1=t \Rightarrow (t^2+1)^2 - 4t^2 \geq 0 \Leftrightarrow (t^2+1)^2 - (2t)^2 \geq 0$

$$\Leftrightarrow (t^2+1-2t)(t^2+1+2t)\geq 0 \Leftrightarrow$$

$$(t-1)^2\cdot(t+1)^2 \geq 0 \Leftrightarrow \left[(t-1)(t+1)\right]^2 \geq 0 \Leftrightarrow$$

$$\left[(a+1-1)\cdot(a+1+1)\right]^2 \geq 0 \quad \left[a\cdot(a+2)\right]^2 \geq 0 \;(A) \Big\} \Rightarrow S\neq\emptyset$$

$(\forall)\, a\in\mathbb{R}$