Descompunem în factori

$x^2+x+1=?$ $a=1\ b=1\ c=1$ $\Delta=b^2-4ac=1^2-4\cdot1\cdot1=1-4=-3$

$\Delta<0 \Rightarrow x^2+x+1$ - nu se descomp. în $\mathbb{R}$

(V1)

$2x^2-x-10=?$ $a=2\ b=-1\ c=-10$ $\Rightarrow \Delta=b^2-4ac=(-1)^2-4\cdot2\cdot(-10)=1+80=81,\ \sqrt{\Delta}=\sqrt{81}=9$

$x_{1,2}=\frac{-b\pm\Delta}{2a}=\begin{cases}\frac{-(-1)+9}{2\cdot2}=\frac{10}{4}=\frac{5}{2}\\ \frac{-(-1)-9}{2\cdot2}=\frac{-8}{4}=-2\end{cases}$

$ax^2+bx+c=0 \Rightarrow x_1, x_2 \dots$

$a\cdot(x-x_1)(x-x_2)=ax^2+bx+c,\ a\neq0$

$2x^2-x-10=2\cdot\left(x-\frac{5}{2}\right)\cdot(x-(-2))=\left(2x-2\cdot\frac{5}{2}\right)\cdot(x+2)=$

$=(2x-5)(x+2)$

(V2)

$2x^2-x-10=2x^2+4x-5x-10=2x\cdot(x+2)-5(x+2)=$

$=(x+2)(2x-5)$

$x^2+x-3=?$ $a=1\ b=1\ c=-3$ $\Delta=b^2-4ac=1^2-4\cdot1\cdot(-3)=1+12=13 \Rightarrow \sqrt{\Delta}=\sqrt{13}$

$x_{1,2}=\frac{-b\pm\sqrt{\Delta}}{2a}=\begin{cases}\frac{-1+\sqrt{13}}{2}\\ \frac{-1-\sqrt{13}}{2}\end{cases}$

$x^2+x-3=\left(x-\frac{-1+\sqrt{13}}{2}\right)\cdot\left(x-\frac{-1-\sqrt{13}}{2}\right)=\left(x+\frac{1-\sqrt{13}}{2}\right)\cdot\left(x+\frac{1+\sqrt{13}}{2}\right)$

$x^2+2=?$ $\quad 1\cdot x^2+0\cdot x+2=0$ $\quad a=1\ b=0\ c=2$

$\Delta=b^2-4ac=0^2-4\cdot1\cdot2=0-8=-8<0$

x^2+2 - nu se descomp. în $\mathbb{R}$

$x^2+4x=?$ $\quad x^2+4x=x(x+4)$

$x^3-9x=x(x^2-9)=x\cdot(x^2-3^2)=x\cdot(x-3)(x+3)$

$x^2+4x+0=0$ $a=1\ b=4\ c=0$

$x^4 + 9 = ?$ $\quad a^2 \pm 2ab + b^2 = (a \pm b)^2 \quad a^2 - b^2 = (a-b)(a+b)$

$x^4 + 9 = (x^2)^2 + 2 \cdot x^2 \cdot 3 + 3^2 - 6x^2 = (x^2+3)^2 - (\sqrt{6}x)^2 =$

$= (x^2 + 3 - \sqrt{6}x) \cdot (x^2 + 3 + \sqrt{6}x) = (x^2 - \sqrt{6}x + 3) \cdot (x^2 + \sqrt{6}x + 3)$

$x^2 - \sqrt{6}x + 3 = 0$
$a = 1 \quad b = -\sqrt{6} \quad c = 3$
$\Delta = b^2 - 4ac = (-\sqrt{6})^2 - 4 \cdot 1 \cdot 3 = 6 - 12 = -6$
$\Delta < 0 \Rightarrow x^2 - \sqrt{6}x + 3$ nu se mai descompune în $\mathbb{R}$

analog: $x^2 + \sqrt{6}x + 3$ Δ

$x^4 + 5x^2 + 4 = ?$

$x^4 + 3x^2 + 4 = (x^2)^2 + 2 \cdot x^2 \cdot 2 + 2^2 - x^2 = (x^2+2)^2 - x^2 =$

$= (x^2 + 2 + x) \cdot (x^2 + 2 - x) = (x^2 + x + 2)(x^2 - x + 2)$

$a = 1 \quad b = 1 \quad c = 2$
$\Delta = b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot 2 = 1 - 8 = -7$
$\Delta < 0 \Rightarrow$ nu se mai descompune în $\mathbb{R}$

$x^2 - 6x - y^2 + 4y + 5 = ?$

$x^2 - 6x - y^2 + 4y + 5 = x^2 - 2 \cdot 3 \cdot x + 3^2 - (y^2 - 2 \cdot 2 \cdot y + 2^2) =$

$= (x-3)^2 - (y-2)^2 = ((x-3) - (y-2)) \cdot ((x-3) + (y-2)) =$

$= (x - 3 - y + 2) \cdot (x - 3 + y - 2) =$

$= (x - y - 1)(x + y - 5)$

$9x^2 - 6x - 4y^2 - 12y - 8 =$

$= (3x)^2 - 2 \cdot 3 \cdot x + 1^2 - ((2y)^2 - 2 \cdot 2 \cdot y \cdot 3 + 3^2) =$

$= (3x-1)^2 - (2y-3)^2 = (3x - 1 - 2y + 3)(3x - 1 + 2y - 3) =$

$= (3x - 2y + 2)(3x + 2y - 4)$

$x^3+2x^2-9x-18=?$

(V1) $x^3+2x^2-9x-18 = x^2(x+2)-9(x+2) = (x+2)(x^2-9) =$
$= (x+2)(x-3)(x+3)$

$18 = 2 \cdot 3^2$ (1 2)

(V2) Scriem divizorii termenului liber:

$D_{18} = \{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$ $(1+1)(2+1) = 2 \cdot 3 = 6$ divizori în $\mathbb{N}$

Calculăm $P(k)$, până când obținem $P(k)=0$, $k \in D_{18}$

$P(1) = 1^3 + 2 \cdot 1^2 - 9 \cdot 1 - 18 = 1+2-9-18 \neq 0$

$P(-1) = (-1)^3 + 2 \cdot (-1)^2 - 9 \cdot (-1) - 18 = -1+2+9-18 \neq 0$

$P(2) = 8+8-18-18 \neq 0$

$P(-2) = (-2)^3 + 2 \cdot (-2)^2 - 9 \cdot (-2) - 18 = -8+8+18-18 = 0$

$P(x) = 1 \cdot x^3 + 2 \cdot x^2 - 9x - 18$ $\quad P(-2)=0$

	1	2	-9	-18
-2	1	0	-9	0

$-2 \cdot 1 + 2 = 0$

$-2 \cdot 0 + (-9) = 0 - 9 = -9$

$-2 \cdot (-9) + (-18) = 18 - 18 = 0$

SCHEMA LUI HORNER

$x^3+2x^2-9x-18 = (x-(-2)) \cdot (1 \cdot x^2 + 0 \cdot x - 9) =$
$= (x+2)(x^2-3^2) = (x+2)(x-3)(x+3)$

$Q(x) = x^3 - 13x - 12$ $\quad \left\{ x^2-1 = x^2-1^2 = (x-1)(x+1) \right.$

(V1)

$Q(x) = x^3 - x - 12x - 12 = x(x^2-1) - 12 \cdot (x+1) =$
$= x \cdot (x-1)(x+1) - 12(x+1) = (x+1) \cdot [x(x-1) - 12] =$
$= (x+1) \cdot (x^2 - x - 12) = (x+1) \cdot (x^2 + 3x - 4x - 12) =$
$= (x+1)(x(x+3) - 4(x+3)) = (x+1)(x+3)(x-4)$

...