Descompunem în factori

$x^4+1=(x^2)^2+2\cdot x^2\cdot 1+1^2-2x^2=(x^2+1)^2-(\sqrt{2}x)^2=(x^2+1-\sqrt{2}x)(x^2+1+\sqrt{2}x)=$

$=(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)$ → nu se mai descompun mai mult:

$a=1 \quad b=-\sqrt{2} \quad c=1$

$x^2-\sqrt{2}x+1=0 \quad \Delta=b^2-4ac=(-\sqrt{2})^2-4\cdot 1\cdot 1=2-4=-2<0$

$x^4+16=(x^2)^2+2\cdot 4\cdot x^2+4^2-8x^2=(x^2+4)^2-(\sqrt{8}\cdot x)^2=$

$[a^2+2ab+b^2=(a+b)^2 \quad a^2-b^2=(a-b)(a+b) \quad \sqrt{8}=\sqrt{4\cdot 2}=2\sqrt{2}]$

$=(x^2+4-2\sqrt{2}x)(x^2+4+2\sqrt{2}x)=$

$=(x^2-2\sqrt{2}x+4)(x^2+2\sqrt{2}x+4)$ Nu se mai descompun deoarece:

$x^2-2\sqrt{2}x+4=0$
$a=1 \quad b=-2\sqrt{2} \quad c=4$
$\Delta=b^2-4ac=(-2\sqrt{2})^2-4\cdot 1\cdot 4=8-16=-8<0$

$x^3-2x^2-x+2=?$

(V1) $x^3-2x^2-x+2=x^2\cdot(x-2)-1\cdot(x-2)=(x-2)(x^2-1)=(x-2)(x^2-1^2)=$
$=(x-2)(x-1)(x+1)$

(V2) $P(x)=x^3-2x^2-x+2$

Scriem divizorii termenului liber: $D_2=\{\pm 1, \pm 2\}$

Calculăm: $P(1)=1^3-2\cdot 1^2-1+2=1-2-1+2=0$

Scriem tabelul:

	1	−2	−1	2
1	1	−1	−2	0

← coeficienții lui x

↳ copiem primul coeficient

$x-(-1)=x+1$

$1\cdot 1+(-2)=1-2=-1$
$1\cdot(-1)+(-1)=-1-1=-2$
$1\cdot(-2)+2=-2+2=0$

$P(x)=(x+1)\cdot(1\cdot x^2-1\cdot x-2)=$
$=(x+1)\cdot(x^2-x-2)=$
$=(x+1)\cdot(x^2-2x+x-2)=$
$=(x+1)(x(x-2)-1(x-2))=$
$=(x+1)(x-2)(x-1)$

$P(x)=1\cdot x^3-2x^2-5x+6$ $\quad D_6=\{\pm1,\pm2,\pm3,\pm6\}$

$P(-1)=(-1)^3-2\cdot(-1)^2-5\cdot(-1)+6=-1-2+5+6=8\neq0$

$P(1)=1^3-2\cdot1^2-5\cdot1+6=1-2-5+6=0$

	1	−2	−5	6
1	1	−1	−6	0

$1\cdot1+(-2)=1-2=-1$

$1\cdot(-1)+(-5)=-1-5=-6$

$1\cdot(-6)+6=0$

$P(x)=(x-1)\cdot(1\cdot x^2-1\cdot x-6)=(x-1)(x^2-3x-6)=(x-1)(x^2+2x-3x-6)=$

$=(x-1)\cdot(x\cdot(x+2)-3(x+2))=(x-1)(x+2)(x-3)$

$P(x)=1x^3+2x^2-5x-6$ $\quad D_6=\{\pm1,\pm2,\pm3,\pm6\}$

$P(1)=1^3+2\cdot1^2-5\cdot1-6=1+2-5-6=3-5-6=-2-6=-8$

$P(2)=2^3+2\cdot2^2-5\cdot2-6=8+8-10-6=16-16=0$

	1	2	−5	−6
2	1	4	3	0

$2\cdot1+2=4$ $\quad 2\cdot4+(-5)=8-5=3$ $\quad 2\cdot3+(-6)=6-6=0$

$P(x)=(x-2)\cdot(1\cdot x^2+4x+3)=(x-2)(x^2+x+3x+3)=$

$=(x-2)\cdot(x(x+1)+3(x+1))=(x-2)(x+1)(x+3)$

$x\cdot(x+2)\cdot(x^2+2x-14)+48=?$ $\quad$ Notăm $x^2+2x=t$

$x\cdot(x+2)\cdot(x^2+2x-14)+48=(x^2+2x)\cdot(x^2+2x-14)+48=t\cdot(t-14)+48=$

$=t^2-14t+48=t^2-6t-8t+48=t(t-6)-8(t-6)=(t-6)(t-8)=$

$=(x^2+2x-6)(x^2+2x-8)=$

$=(x-(-1+\sqrt{7}))\cdot(x-(-1-\sqrt{7}))\cdot(x^2+2x-8)=$

$=(x+1-\sqrt{7})\cdot(x+1+\sqrt{7})\cdot(x^2+2x-8)=$

$=(x+1-\sqrt{7})(x+1+\sqrt{7})(x+4)(x-2)$

$1\cdot x^2+2\cdot x+(-6)=0$

$a=1 \quad b=2 \quad c=-6$

$\Delta=b^2-4ac=4-4\cdot1\cdot(-6)=4+24=28$

$\sqrt{\Delta}=\sqrt{4\cdot7}=2\sqrt{7}$

$x_{1,2}=\dfrac{-2\pm2\sqrt{7}}{2\cdot1}=\begin{cases}\dfrac{-2+2\sqrt{7}}{2}=\dfrac{2\cdot(-1+\sqrt{7})}{2}=-1+\sqrt{7}\\ \dfrac{-2-2\sqrt{7}}{2}=\dfrac{2\cdot(-1-\sqrt{7})}{2}=-1-\sqrt{7}\end{cases}$

$x^2+2x-8=x^2+4x-2x-8=$

$=x(x+4)-2(x+4)=$

$=(x+4)\cdot(x-2)$

$\sqrt{6-4\sqrt{2}} = ?$ $\qquad \sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+\sqrt{A^2-B}}{2}} \pm \sqrt{\frac{A-\sqrt{A^2-B}}{2}}$

$\sqrt{6-4\sqrt{2}} = \sqrt{6-\sqrt{16\cdot 2}} = \sqrt{6-\sqrt{32}} = \sqrt{\frac{6+\sqrt{6^2-32}}{2}} - \sqrt{\frac{6-\sqrt{6^2-32}}{2}} =$

$= \sqrt{\frac{6+\sqrt{4}}{2}} - \sqrt{\frac{6-\sqrt{4}}{2}} = \sqrt{\frac{6+2}{2}} - \sqrt{\frac{6-2}{2}} = \sqrt{4} - \sqrt{2} = 2-\sqrt{2}$

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