

Descompuneri in factori

1) Scoaterea factorului comun:

$$\begin{aligned}
 24x \cdot yx \cdot z \cdot y^2 - 12x \cdot yx \cdot yx \cdot z^3 + 18x \cdot y^2 \cdot z^2 &= \begin{cases} 24 = 2^3 \cdot 3 \\ 12 = 2^2 \cdot 3 \\ 18 = 2 \cdot 3^2 \\ (24, 12, 18) = 2 \cdot 3 = 6 \end{cases} \\
 = 24x^2 y^3 z - 12x^3 y^2 z^3 + 18x y^2 z^2 &= \\
 = 6 \cdot x \cdot y^2 \cdot z \cdot (4xy - 2x^2 \cdot z^2 + 3z) &=
 \end{aligned}$$

$$\begin{aligned}
 5 \cdot x \cdot 5 \cdot x \cdot 2y^2 - 15x^3 \cdot 3 \cdot y + 25x^2 y &= 50x^2 y^2 - 45x^3 y + 25x^2 y = \\
 = 5x^2 y \cdot (10y - 9x + 5) &= \begin{cases} \begin{array}{r} +5 \overline{) 5} \rightarrow 5 \\ 15 \overline{) 5} \rightarrow 5 \\ 3 \overline{) 3} \rightarrow 1 \end{array} & \begin{array}{r} 27 \overline{) 3^2} \rightarrow 3 \\ 3 \overline{) 3} \rightarrow 1 \end{array} & \begin{array}{r} 12 \overline{) 2^2} \rightarrow 2 \\ 3 \overline{) 3} \rightarrow 1 \end{array} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 3\sqrt{75} x^2 y - 2\sqrt{27} xy^2 + 4\sqrt{12} xy &= 3 \cdot 5\sqrt{3} x^2 y - 2 \cdot 3\sqrt{3} xy^2 + 4 \cdot 2\sqrt{3} xy = \\
 = \sqrt{3} x \cdot y \cdot (15x - 6y + 8) &= \begin{cases} -2 + ? = 2 \Rightarrow ? = 2 + 2 \Rightarrow ? = 4 \\ -2 + ? = 0 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 2x^m \cdot y^{m+2} - 4x^{m-1} \cdot y^m - 8x^{m+3} \cdot y^{n-2} &= \\
 = 2 \cdot x^{m-1} \cdot x^1 \cdot y^{m-2} \cdot y^4 - 4 \cdot x^{m-1} \cdot y^{m-2} \cdot y^2 - 8 \cdot x^{m-1} \cdot x^4 \cdot y^{m-2} &= \\
 = 2 \cdot x^{m-1} \cdot y^{m-2} \cdot (x \cdot y^4 - 2y^2 - 4x^4) &= \begin{cases} -4 + ? = 5 \Rightarrow ? = 5 + 4 \\ -3 + ? = 2 \Rightarrow ? = 2 + 3 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 x^{m+5} \cdot y^{n-3} + x^{n-4} \cdot y^{m+2} - x^{m-2} \cdot y^{n-1} &= \\
 = x^{m-4} \cdot y^{n-3} \cdot x^9 + x^{m-4} \cdot y^{n-3} \cdot y^5 - x^{m-4} \cdot x^2 \cdot y^{m-3} \cdot y^2 &= \\
 = x^{m-4} \cdot y^{n-3} \cdot (x^9 + y^5 - x^2 y^2) &= \begin{cases} a^2 - b^2 = (a+b)(a-b) \\ a^2 + 2ab + b^2 = (a+b)^2 \end{cases}
 \end{aligned}$$

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$$\begin{aligned}
 x^2 - x - 6 &= \underbrace{x^2 + 2x - 3x - 2 \cdot 3}_{=} = x(x+2) - 3 \cdot (x+2) = \\
 &= (x+2) \cdot (x-3)
 \end{aligned}$$

$$x^2 + 5x + 6 = \overbrace{x^2 + 2x}^{x(x+2)} + \overbrace{3x + 6}^{3(x+2)} = x(x+2) + 3(x+2) = (x+2)(x+3)$$

$$x^2 - 5x + 4 = \overbrace{x^2 - 1 \cdot x}^{x(x-1)} - \overbrace{4 \cdot x + 4}^{4(x+1)} = x(x-1) - 4(x+1) = (x-1)(x-4)$$

$$x^2 - 7x - 8 = \overbrace{x^2 - 4x}^{x(x-4)} + \overbrace{-2x - 8}^{-2(x+4)} = (x-4)(x+2)$$

$$x^2 - x - 2 = \overbrace{x^2 + x}^{x(x+1)} - \overbrace{2x + 2}^{2(x+1)} = x(x+1) - 2(x+1) = (x+1)(x-2)$$

$$2x^2 + x - 3 = ?$$

Fie $\underbrace{a, b, c}_{\text{coeficienți}} \in \mathbb{R}, x \in \mathbb{R}, a \neq 0$

↳ coeficienți

$$ax^2 + bx + c = 0$$

Calculăm un număr, $\Delta = b^2 - 4ac \rightarrow$ vom avea 3 variante:

$$i) \Delta < 0$$

↓

$$ax^2 + bx + c \Rightarrow$$

nu se poate descompune în $\mathbb{R}$

$$ii) \Delta > 0$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

↓

$$ax^2 + bx + c = a \cdot (x - x_1)(x - x_2)$$

$$iii) \Delta = 0$$

$$x_1 = x_2 = \frac{-b}{2a}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_1) = a(x - x_1)^2$$

$$2x^2 + x - 3 = ?$$

$$a = 2$$

$$b = 1$$

$$c = -3$$

$$\Delta = b^2 - 4ac = 1^2 - 4 \cdot 2 \cdot (-3) = 1 + 24 = 25$$

$$\sqrt{\Delta} = \sqrt{25} = 5$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-1 + 5}{2 \cdot 2} = \frac{4}{4} = 1$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-1 - 5}{2 \cdot 2} = \frac{-6}{4} = -\frac{3}{2}$$

$$2x^2 + x - 3 = 2 \cdot (x - 1) \cdot (x - (-\frac{3}{2})) = 2 \cdot (x - 1) \cdot (\frac{x}{1} + \frac{3}{2}) =$$

$$\overset{1}{=} 2 \cdot (x - 1) \cdot \frac{2x + 3}{2 \cdot 1} = (x - 1)(2x + 3)$$

$$2x^2 + x - 3 = 2 \cdot (x-1) \cdot \left(x + \frac{3}{2}\right) = (x-1) \cdot \left(2x + \frac{3}{1}\right) = (x-1)(2x+3)$$

$$\begin{array}{l} 3x^2 - x + 10 = ? \\ + \underbrace{3 \cdot x^2}_a + \underbrace{(-1)x}_b + \underbrace{10}_c = ? \end{array} \left\{ \begin{array}{l} a=3 \quad \Delta = b^2 - 4ac \\ b=-1 \Rightarrow \Delta = (-1)^2 - 4 \cdot 3 \cdot 10 = 1 - 120 = -119 \\ c=10 \end{array} \right. \sqrt{\Delta} = \sqrt{-119} \notin \mathbb{R} \Rightarrow$$

$3x^2 - x + 10$ nu se descompune în $\mathbb{R}$

Atentie: dacă $a < 0 \Rightarrow \sqrt{a} \notin \mathbb{R}$

$$\text{Dacă } \sqrt{-4} = 2 \Rightarrow 2^2 = -4 \Rightarrow 4 = -4 \text{ (F)}$$

$$\sqrt{-4} = -2 \Rightarrow (-2)^2 = -4 \Rightarrow +4 = -4 \text{ (F)}$$

$$\sqrt{-4} \notin \mathbb{R}$$

$$\begin{array}{r} 121 \\ 11 \overline{) 121} \\ \underline{11} \\ 11 \\ \underline{11} \\ 0 \end{array} > 11$$

$$\begin{array}{l} 3x^2 - x - 10 = ? \\ + \underbrace{3 \cdot x^2}_a + \underbrace{(-1)x}_b + \underbrace{(-10)}_c = ? \end{array} \quad \begin{array}{l} a=3 \quad \Delta = b^2 - 4ac \\ b=-1 \quad \Delta = (-1)^2 - 4 \cdot 3 \cdot (-10) \\ c=-10 \quad \Delta = 1 + 120 = 121 \Rightarrow \sqrt{\Delta} = \sqrt{121} = 11 \end{array}$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-1) + 11}{2 \cdot 3} = \frac{1+11}{6} = \frac{12}{6} = 2$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-1) - 11}{2 \cdot 3} = \frac{1-11}{6} = \frac{-10}{6} = -\frac{5}{3}$$

$$3x^2 - x - 10 = 3 \cdot (x-2) \cdot \left(x - \left(-\frac{5}{3}\right)\right) = 3(x-2)\left(x + \frac{5}{3}\right) =$$

$$= (x-2) \cdot \left(3x + \frac{5}{1}\right) = (x-2) \cdot (3x+5)$$

$$\begin{array}{l} x^2 + 10x + 25 = ? \\ 1 \cdot \underbrace{x^2}_a + \underbrace{10x}_b + \underbrace{25}_c = ? \end{array} \left\{ \begin{array}{l} \Delta = b^2 - 4ac = 100 - 4 \cdot 25 = 100 - 100 = 0 \\ x_1 = x_2 = \frac{-10}{2 \cdot 1} = -5 \\ x^2 + 10x + 25 = 1 \cdot (x+5)(x+5) = (x+5)^2 \\ \downarrow \\ (x-(-5))(x-(-5)) \end{array} \right.$$

$$x^2 + 10x + 25 = x^2 + 2 \cdot 5x + 5^2 = (x+5)^2$$